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Models of repairable spare parts consumption based on various constraint conditions

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Abstract. Considering the repaired as good as new, repaired as old and imperfect maintenance of partially repairable spare parts and choosing the failure rate and maintenance cost as the constraint conditions, this paper established forecasting models of repairable spares parts consumption based on calculating the average usage time under different usage and repairement stages of spares parts, applying equipment reliability theory and stochastic process theory. At last, an example was taken to illustrate the applicability of these models. These models provide sufficient scientific basis for choosing repairable spare parts application, storage and supply amount reasonably, and have the vital important guiding significance.

Key words. Repairable spare parts, failure rate, maintenance cost, spare parts consumption.
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1. Introduction

At present, according to the characteristics of the equipment and the characteristics of maintenance, although the forecasting methods of spare parts consumption of the equipment have been studied in depth, the methods still can't meet the actual needs of equipment management work. The historical consume data of equipment spare parts are often very limited and more limited to forecasting methods. Many complex cases of consumption forecast problems of equipment spare parts also need to be studied. In addition, restrictions of the existing model are too harsh, which results in a limited scope of application.

Many of the forecasting models and methods proposed by the US military have high reference value for the research of the equipment consumption forecasting meth-

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ods. The US military often uses statistical methods in conjunction with expert decision-making, and the parameters in the model must be estimated by using expert decision-making methods that can be used to assist the analyst in systematically thinking about the problem to ensure that the impact of each element on the consumption of spare parts can be fully taken into account [1]; The US established

the Boke–Jenkins model, which was based on the historical data of aircraft spare parts [2], [3]. By comparison, it is found that the prediction accuracy of the model is more accurate than the linear regression model. The basic types of the Boke–Jenkins model include: moving average model, autoregressive model and autoregressive-moving average model and the autoregressive-moving average model is most commonly used in the three models. According to the characteristics of consumption data of different equipment spare parts, the US military uses the exponential smoothing model and the moving average method [4], [5] to forecast the consumption of different ship spare parts. Other authors have also done research on spare parts consumption forecasting [6].Through the analysis of the literature on the consumption rules of equipment spare parts at home and abroad, it can be found that the scholars in this field mainly study the consumption of spare parts, and the research on the consumption forecast method of repairable spare parts is rare.

Some scholars have studied the demand forecasting methods for repairable spare parts, but there are still essential differences in the consumption and demand of repairable spare parts. Repairable spare parts are the spare parts whose original function can be restore by using economical and feasible technical means after malfunction or damage. At present, the number of the repairable spare parts of formed unit equipment is large, which directly relates to the use of equipment availability, and occupies a part of the spare parts protection funds. The maintenance of repairable spare parts can improve equipment availability and, on the other hand, save maintenance resources and costs. But in practice, repairable spare parts of equipment can't be unlimited cycle repair use, and the vast majority is limited repairable spare parts. According to the maintenance costs, failure and other factors, there are a certain degree of retirement and spare parts consumption.

In this paper, the unit failure rate and maintenance costs are considered as a constraint to study the forecast method of repairable spare parts consumption. The forecasting models of repairable spare parts consumption are relative with many failure, but other forecasting models of spare parts consumption in other papers are only relative with one failure. So it is necessary to do research on the forecasting models of repairable spare parts consumption based on various constraint conditions.

2. Formulation of the problem

Equipment unit after failure is repaired. If the state of the unit is the basic same as the state before fault, we call it "maintenance as old"; if the state of the unit reaches the state of the new product, we call it "maintenance as new"; if the state of the unit can't reach the state of new products, but can be some improvement compared with the state before the maintenance, the unit reaches between a new state and the old, we call it "incomplete maintenance".

After the first failure of unit occurred during the period of use, the unit's service age can be backward for a while through the repair of the maintenance period of time; after the second failure of unit occurred, the unit's service age can be backward for a while again through the repair of the maintenance period of time; and so on, until the unit maintenance costs incurred by the maintenance is close to or reached the specified value, when the next failure is no longer used and repaired, repairable parts must be carried out, which results in a spare parts consumption.

3. Mathematical model

The function $f(t)$ is the failure probability density function of the unit after starting from the new product; $R(t)$ is the reliability function of the unit after starting from the new product; $f_j(t_j)$ is the probability density function of the unit between the $j-1$th time point of maintenance and the j th time point of maintenance; μ is the retreat factor of service age , that is, the repairable maintenance of the unit makes the repairable maintenance interval go back μ times averagely $0 \le \mu \le 1$; we determine whether the unit after the failure of retirement, according to its failure rate and maintenance costs, and the failure rate of the critical value is $\lambda(T_0) = \lambda_0$ (critical point), $T_0 = \lambda^{-1}(\lambda_0)$; Q is the maintenance cost , and q is the cost of repairing the unit on average.

Then, based on the limited value of the maintenance cost of the unit and the average cost of a repair, the limited value of the average maintenance number of the unit is

$$N = \left\lfloor \frac{Q}{q} \right\rfloor \tag{1}$$

In equation (1), $\lfloor\rfloor$ represents a round-down.

Therefore, the unit's scrapping is divided into two cases:

(1) When the actual repair number of the unit is less than N, before a failure occurs, the corresponding failure rate has exceeded a certain value, which causes the unit scrapped.

(2) When the actual repair number of the unit is N, after the Nth fix, regardless of whether the corresponding failure rate exceeds the specified value, the unit, which works to the next failure, must be scrapped.

(A) When the situation is (1), the probability that the number of experienced maintenance is 0 before the unit is scrapped is

$$P_0 = P(T_1 \ge T_0) = 1 - \int_0^{T_0} f_1(t_1)\,dt_1 = \int_{T_0}^{+\infty} f_1(t_1)\,dt_1 \tag{2}$$

When the number of experienced maintenance is 0 before the unit is scrapped, the average working time of the unit is

$$\theta_0 = \frac{\int_{T_0}^{+\infty} t_1 f_1(t_1)\,dt_1}{R_1(T_0)} \tag{3}$$

In equations (2) to (3): $f_1(t_1) = f(t_1)\ R_1(t) = 1 - \int_0^t f_1(t_1)\,\mathrm{d}t_1$.

The probability that the number of experienced maintenance is 1 before the unit is scrapped is

$$P_1 = P\left[(1-u)T_1 + T_2 \geq T_0, T_1 < T_0\right] = \int_0^{T_0}\int_{T_0-(1-u)t_1}^{+\infty} f_1(t_1) f_2(t_2)\,\mathrm{d}t_1\mathrm{d}t_2 \quad (4)$$

When the number of experienced maintenance is 1 before the unit is scrapped, the average working time of the unit is

$$\theta_1 = \frac{\int_0^{T_0} t_1 f_1(t_1)\,dt_1}{1-R_1(T_0)} + \int_0^{T_0}\int_{T_0-(1-u)t_1}^{+\infty} \frac{f_1(t_1)}{1-R_1(T_0)}\frac{t_2 f_2(t_2)}{R_2\left[T_0-(1-u)t_1\right]}\mathrm{d}t_1\mathrm{d}t_2 \quad (5)$$

In equations (4) to (5): $f_2(t_2) = \frac{f[t_2+(1-u)t_1]}{R[(1-u)t_1]}??R_2(t) = 1 - \int_0^t f_2(t_2)\,\mathrm{d}t_2$.

The probability that the number of experienced maintenance is 2 before the unit is scrapped is

$$P_2 = P\left[(1-u)(T_1+T_2) + T_3 \geq T_0, T_1 < T_0, (1-u)T_1 + T_2 < T_0\right]$$

$$= \int_0^{T_0}\int_0^{T_0-(1-u)t_1}\int_{T_0-(1-u)(t_1+t_2)}^{+\infty} f_1(t_1) f_2(t_2) f_3(t_3)\,\mathrm{d}t_1\mathrm{d}t_2\mathrm{d}t_3 \quad (6)$$

When the number of experienced maintenance is 2 before the unit is scrapped, the average working time of the unit is

$$\theta_2 = \frac{\int_0^{T_0} t_1 f_1(t_1)\,dt_1}{1-R_1(T_0)} + \int_0^{T_0}\int_0^{T_0-(1-u)t_1} \frac{f_1(t_1)}{1-R_1(T_0)}\frac{t_2 f_2(t_2)}{1-R_2\left[T_0-(1-u)t_1\right]}\mathrm{d}t_1\mathrm{d}t_2 +$$

$$\int_0^{T_0}\int_0^{T_0-(1-u)t_1}\int_{T_0-(1-u)(t_1+t_2)}^{+\infty} \frac{f_1(t_1)}{1-R_1(T_0)}\frac{f_2(t_2)}{1-R_2\left[T_0-(1-u)t_1\right]}\frac{t_3 f_3(t_3)}{R_3\left[T_0-(1-u)(t_1+t_2)\right]}\mathrm{d}t_1\mathrm{c} \quad (7)$$

In equations (6) to (7): $f_3(t_3) = \frac{f[t_3+(1-u)(t_1+t_2)]}{R[(1-u)(t_1+t_2)]}??R_3(t) = 1 - \int_0^t f_3(t_3)\,\mathrm{d}t_3$.

And so on, the probability that the number of experienced maintenance is $n\,(n \geq 3)$ before the unit is scrapped is

$$P_n = P\left[(1-u)\sum_{i=1}^{n} T_i + T_{n+1} \geq T_0, T_1 < T_0, (1-u)\sum_{i=1}^{j-1} T_i + T_j < T_0\,(j = 2,3,4,\cdots,n)\right]$$

$$= \int_0^{T_0}\int_0^{T_0-(1-u)t_1}\cdots\int_0^{T_0-(1-u)\sum_{i=1}^{n-1} t_i}\int_{T_0-(1-u)\sum_{i=1}^{n} t_i}^{+\infty}\prod_{i=1}^{n+1}\left[f_i(t_i)\,\mathrm{d}t_i\right] \quad (8)$$

When the number of experienced maintenance is $n\,(3 \leq n \leq N-1)$ before the unit is scrapped, the average working time of the unit is

$$R = X + Y \tag{9}$$

In equations (8) to (9):
$f_{n+1}(t_{n+1}) = \frac{f\left[t_{n+1}+(1-u)\sum_{i=1}^{n} t_i\right]}{R\left[(1-u)\sum_{i=1}^{n} t_i\right]}$, $R_{n+1}(t) = 1 - \int_0^t f_{n+1}(t_{n+1})\,\mathrm{d}t_{n+1}$.

(B) When the situation is (2), the probability that the number of experienced maintenance is N is

$$P_N = 1 - \sum_{i=0}^{N-1} P_i \tag{10}$$

When the number of experienced maintenance is N, the average working time of the unit is

$$R = 1 \tag{11}$$

In equation (11) $f_{N+1}(t_{N+1}) = \frac{f\left[t_{N+1}+(1-u)\sum_{i=1}^{N} t_i\right]}{R\left[(1-u)\sum_{i=1}^{N} t_i\right]}$ $R_{N+1}(t) = 1 - \int_0^t f_{N+1}(t_{N+1})\,\mathrm{d}t_{N+1}$.

(C) The forecast of spare parts consumption

According to equations (1) to (11), the total average working time of the unit is

$$\theta = \sum_{i=0}^{N-1} P_i\theta_i + \left(1 - \sum_{i=0}^{N-1} P_i\right)\theta_N \tag{12}$$

Therefore, when the number of equipment is W and the unit's single-use number is L, the estimated number of spare parts consumption of units in the next T time is

$$y = \frac{WLT}{\theta} = \frac{WLT}{\sum_{i=0}^{N-1} P_i\theta_i + \left(1 - \sum_{i=0}^{N-1} P_i\right)\theta_N} \tag{13}$$

4. Analysis of an example

A system of units is equipped with a certain type of equipment and the number of equipment is 10. A type of unit failure of equipment in the conditions can be repaired within the allowed scope. The basic information such as the number of units used for the single, the limited value of the maintenance cost, the retreat factor of service age after maintenance and the failure probability density function of the unit after starting from the new product. is shown in Table 1. The average cost of a repairable maintenance of the unit is 7000 yuan. We determine whether the unit is scrapped or not after the failure, according to the critical value of the failure rate ($\lambda(T_0) = 0.04/\mathrm{h}$) or maintenance costs of achieving the specified value. Please predict the number of consumed spare parts when the work time of each equipment is 1000h on average.

Table 1. Basic information of a certain type of unit

Serial number	The number of units	The limited cost of maintenance	The retreat factor of service age	The failure probability density function since the beginning
1	4	45000	0.4	$f(t) = \frac{2t}{1500^2} e^{-\left(\frac{t}{1500}\right)^2}$

The number of equipment is $W = 10$; the number of single is $L = 4$; the retreat factor of service age is $\mu = 0.4$; the failure probability density function since the beginning follows the Weibull distribution.

According to equation (1), the limited value of the average maintenance number of the unit is

$$N = \left\lfloor \frac{Q}{q} \right\rfloor = \left\lfloor \frac{45000}{7000} \right\rfloor = 6$$

The unit is incomplete maintenance every time. According to equations (2) to (11), programming calculations can be done. Then, the average working time trend of the unit when the number of repairs is different and the probability of change about the number of repairs that may be experienced before the unit is scrapped are obtained, as shown in Fig.1 and Fig.2.

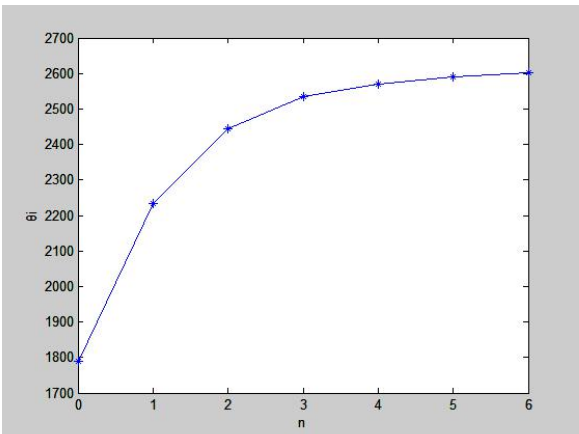

Fig. 1. Average working time trend of the unit when the number of repairs is different

As can be seen from Fig.1, with number of repair increasing, the unit average working time monotonically increases; as can be seen from Fig.2, when the number of repairs that may be experienced before the unit is scrapped is 3, the probability is the highest; when the number of repairs is less than 3, the value of probability shows a monotonically increasing trend; when the number of repairs is more than 3, the value of probability shows a monotonically decreasing trend.

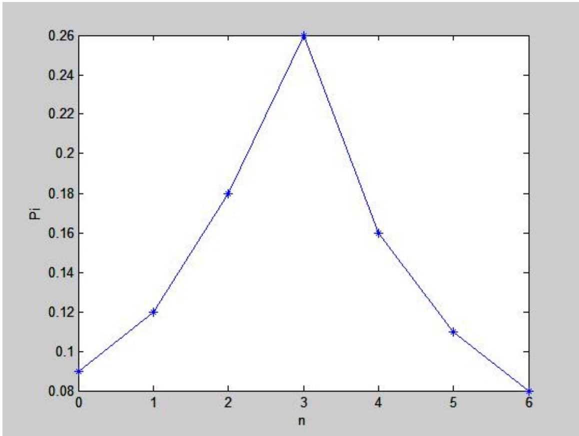

Fig. 2. robability of change about the number of repa-irs that may be experienced before the unit is scrapped

According to equation (12), the total average working time of the unit is

$$\theta = \sum_{i=0}^{N-1} P_i\theta_i + \left(1 - \sum_{i=0}^{N-1} P_i\right)\theta_N = 2432\text{h}$$

According to equation (13), it is estimated that the total amount of spare parts consumed by the unit on all the equipment in the next 1000 hours is

$$y = \frac{10 \times 4 \times 1000}{2432} = 16.4$$

According to the estimated amount of spare parts consumption, combining with the inventory of spare parts standards, we can prepare the batch supply program of the repairable spare parts.

5. Conclusion

In this paper, the forecasting model of repairable spare parts is established by using the reliability theory and the stochastic process theory, which is based on the failure rate and maintenance cost of the repairable spare parts. The forecasting methods of repairable spare parts consumption based on various constraint conditions are the highlight of innovation in this paper. They are very valuable to spare parts support. With reference to the forecasting model, we can further consider the failure rate, maintenance cost and repair method of the unit at regular maintenance time, and give the forecasting method of repairable spare parts, so as to provide theoretical basis for the equipment support department to develop scientific and reasonable repairable spare parts support program.

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